of the jet calculated from (10) on the basis of (9) and (11) and shown in Fig. 1. For small values of n, which correspond to a substantial jet width, it becomes incorrect to consider the problem within the framework of boundary-layer theory.

In calculating the width one takes the two real symmetrical roots $\pm\alpha$ of $\varphi(\zeta) = 0$; a nonlinear liquid also shows a tendency for the ejection capacity to increase as n decreases, along with the change in the geometry [1].

NOTATION

x, y, longitudinal and transverse coordinates; u, v, longitudinal and transverse velocities; ρ, density; τ, shear stress; n, rheological parameter characterizing the non-Newtonian behavior; k, consistency measure; K_0, momentum; ξ, η, Mises variables [formula (4)]; b(x), jet boundary; u_m, maximum velocity in the section.

LITERATURE CITED

1. Z. P. Shul'man, Convective Heat and Mass Transfer for a Rheologically Complex Liquid [in Russian], Énergiya, Moscow (1975), pp. 120-127.
2. L. G. Loitsyanskii, Laminar Boundary Layers [in Russian], Gosizdat. Fiz.-Mat. Literatury, Moscow (1962), pp. 23-26, 39-42.

RESISTANCE OF A BODY OF ROTATION WITH A CENTRAL HOLE IN A SUPERSONIC FLOW

V. V. Tsymbalov

UDC 533.6.001.2

Simulation is applied to a body with conical inlet and outlet to determine the dependence of the resistance on the geometrical characteristics and the Mach number of the supersonic flow.

A body of rotation with central flow belongs to the class for which the aerodynamic features in supersonic flow are largely determined by the variations in the shock-wave structure as the flow speed varies. We have examined the resistance law for such a body for Mach numbers in the unperturbed flow in the range $1.2 \leq M_\infty \leq 6$ (here and subsequently, subscript ∞ denotes a parameter in the unperturbed flow). The study is numerical by means of Godunov's nonstationary difference scheme [1], which is used with an algorithm for constructing oblique-angle cells and a system of Euler equations written in a cylindrical coordinate system. The standard boundary conditions are used for the incident unperturbed flow, at the symmetry axis, and at the solid surfaces; at the other open surfaces we use the conditions for zero values of the derivatives of the gasdynamic parameters along the normals to these surfaces. The distance from the surface of the body to the boundaries of the working region was selected during the numerical experiments, along with the nonuniformity in the distribution of the nodal lines in this region.

The body (Fig. 1a) is a cylinder of diameter D and of length L = 1.5 or 2.25D with a hole of diameter $d \leq 0.9D$ with conical inlet and outlet. The cone angles θ_1 at the inlet were 2, 20, and 90°, while those at the outlet were θ_2 = 20, 26, and 90° (the form $\theta_1 = \theta_2 = 90°$ is a cylinder with a central hole and no sharp edges at the inlet and outlet). For $\theta_1 \neq 90°$ we consider the forms with sharp and blunt edges at the inlet: $d_1 = D$; $d_1 = 0.95D$, where d_1 is the diameter at the leading end section of the cylinder, which determines the degree of blunting of the edges. The edge blunting was taken as zero at the exit from the channel for $\theta_2 \neq 90°$.

Parts b and c of Fig. 1 show the shockwave structures (lines of constant pressure P/P_∞) as found near a body whose geometry was represented by the following set of parameters: d = 0.8D; d_1 = D; L = 1.5D; $\theta_1 = \theta_2 = 20°$ for the case M_∞ = 2 (b) and M_∞ = 6 (c). It is evident

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 3, pp. 405-408, March, 1982. Original article submitted June 14, 1981.

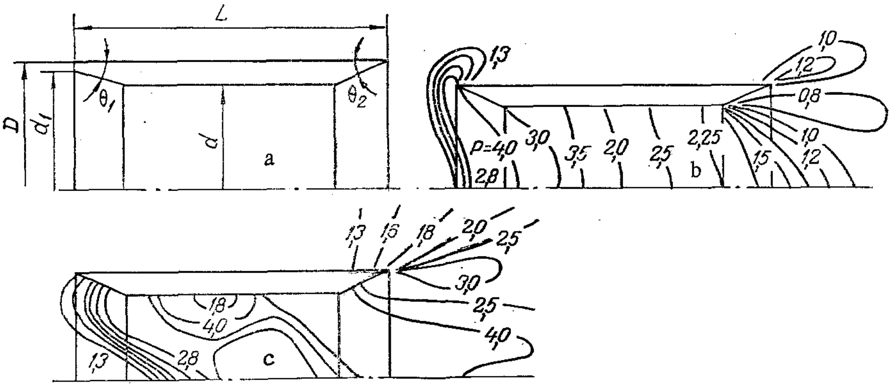

Fig. 1. Cylinder with hole (a) and distribution of isobars in the working region for $M_\infty = 2$ (b) and $M_\infty = 6$ (c) for a body with $d/D = 0.8$; $L/D = 1.5$; $d_1 = D$; $\theta_1 = \theta_2 = 20°$.

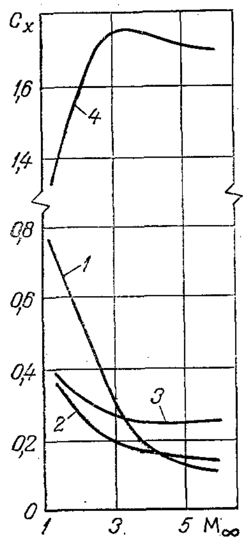

Fig. 2. Profile resistance for a cylinder with a central hole: 1) $d/D = 0.8$; $L/D = 1.5$; $d_1 = D$; $\theta_1 = 20°$; 2) $d/D = 0.8$; $L/D = 2.25$; $d_1/D = 0.95$; $\theta_1 = 2°$; $\theta_2 = 26°$; 3) resistance law of [2]; 4) resistance of a cylinder (wave and bottom resistances) with flat ends in the absence of a central hole [3].

that a head shock wave arises ahead of the body at the lower Mach number and which intersects the symmetry axis in an irregular fashion. This flow state can be characterized as that with flow blockage in the flow part of the body. It is characterized by elevated resistance to the motion of the body and by the presence of a closed subsonic flow region behind the head shock wave. For $M_\infty = 6$, the shock wave is displaced into the body. Also, the intersection with the symmetry axis is regular, and there is a region of supersonic flow behind the shock wave (here the strong smearing of the weak shock waves means that the shock wave reflected from the symmetry axis is not visible in the flow field shown in Fig. 1c).

The fall from $M_\infty = 6$ to $M_\infty = 2$ is accompanied by an increase in the intensity of the head shock wave and a displacement of the point of intersection with the symmetry axis towards the leading edge. A similar effect arises from increasing θ_1. If the angle is small ($\theta_1 = 2°$), the gas penetrating into the body is only slightly perturbed. The intensity of the shock waves propagating in the flow part of the body increases with θ_1, and this is accompanied by an increase in the resistance (note that in this range of M_∞, $\theta_1 = 2°$ and $d = 0.9D$ did not give rise to flow blocking).

Figure 2 shows the M_∞ dependence of the coefficient C_x for the profile resistance at the instant of establishment of the flow (as assigned to $\pi D^2/4$). For θ_1 small, the variation of C_x with Mach number is much less than that for large angles. For $\theta_1 = 20°$, there is a marked increase in C_x as the Mach number decreases below $M_\infty = 4$, which is due to the presence of a head shock wave detached from the body (here possible hysteresis effects were not detected within the framework of the model). Figure 2 gives for comparison data for the resistance of a body without a central hole. Curve 3 corresponds to the resistance law of [2],

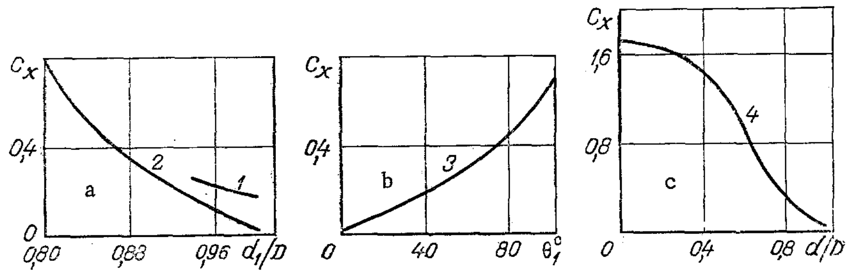

Fig. 3. Effects on the resistance from blunting the inlet edges into the hole (a), the cone angle at the entry to the hole (b), and the diameter of the flow part (c): 1) $d/D = 0.8$; $L/D = 1.5$; $\theta_1 = 20°$; $\theta_2 = 26°$: $M_\infty = 4$; 2) $d/D = 0.8$; $L/D = 2.25$; $\theta_1 = 2°$; $\theta_2 = 26°$; $M_\infty = 6$; 3) $d/D = 0.8$; $L/D = 2.25$; $d_1/D = 1$; $\theta_2 = 20°$; $M_\infty = 6$; 4) $d_1/D = 0.95$; $L/D = 1.5$; $\theta_1 = 20°$; $\theta_2 = 26°$; $M_\infty = 6$.

while curve 4 reproduces the wave and bottom resistances of a cylinder with flat ends [3].

Parts a–c of Fig. 3 give certain results characterizing the dependence of C_x on the geometrical parameters for $M_\infty = 4$ and 6. Here the variable parameters were the diameter d of the hole together with the diameter d_1 and the cone angle θ_1 of the entrance part.

The data of parts a and b of Fig. 3 show that the replacement of a sharp edge by a blunt one for $d_1 = 0.95D$ (curve 1) causes C_x to increase by about 30% for $M_\infty = 4$. Also, C_x is increased by more than a factor 2.5 for $M_\infty = 6$ (curve 2). The absence of a conical inlet ($\theta_1 = 90°$) causes C_x to increase considerably (by more than a factor 5 for $M_\infty = 6$) for $d/D = 0.8$ and $L/D = 1.5$ by comparison with the form of the same body with sharp edges and $\theta_1 = \theta_2 = 20°$ (curves 2 and 3). Variation in the cone angle of the exit part of the channel in the range $20° \leqslant \theta_2 \leqslant 90°$ has virtually no effect on the resistance, at least for $d/D = 0.8$.

Figure 3c shows the effects of d on the resistance in terms of $C_x(d/D)$ for $M_\infty = 6$ for a body with the geometrical characteristics $d_1 = 0.95D$; $L = 1.5D$; $\theta_1 = 20°$; $\theta_2 = 26°$; the resistance coefficient increases as d decreases and tends to the corresponding value for a cylinder with a flat end (here for this configuration a body with $d = 0$ is a cylinder whose leading end has a funnel in the form of a truncated conical recess of depth $d_1/(2 \tan \theta_1)$. The calculations also showed that there is a considerable increase in the time required to attain a steady state of flow as d/D decreases. The maximum time was 3 h with a BÉSM-6 computer with a net containing 2500-3000 nodes in the absence of oscillations due to the presence of the funnel at the leading end. For $d/D = 0.8$, the computation on one form required about 1 h of machine time.

LITERATURE CITED

1. S. K. Godunov (ed.), Numerical Solution of Multidimensional Problems in Gas Dynamics [in Russian], Nauka, Moscow (1976).
2. F. R. Gantmakher and L. M. Levin, Theory of the Flight of Unguided Rockets [in Russian], Fizmatgiz, Moscow (1959).
3. K. P. Petrov, Rocket Aerodynamics [in Russian], Mashinostroenie, Moscow (1977).